Pixel resolution control in numerical reconstruction of digital holography

Lingfeng Yu and Myung K. Kim

Deptment of Physics, University of South Florida, Tampa, Florida 33620

Received November 8, 2005; revised December 19, 2005; accepted January 10, 2006; posted January 12, 2006 (Doc. ID 65891)

A new method for resolution control in numerical reconstruction of digital holography is proposed. The wave field on a tilted or vertical plane can be reconstructed without being subject to the minimum object-to-hologram distance requirement, and the pixel resolution can be easily controlled by adjusting the position of a transitional plane. The proposed method solves the problem of pixel resolution control for small object-to-hologram distances and is especially useful for multicolor, multiwavelength digital holography and metrological applications. Experimental results are presented to verify the idea. © 2006 Optical Society of America

OCIS codes: 090.1760, 100.3010, 090.0090.

Digital holography (DH) is a fast-growing research subject that has drawn increasing attention.[1] It is very important to control the pixel resolution in numerical reconstruction of DH in many applications such as automatic focus and deformation tracking,[2] multicolor DH,[3,4] wavelength-scanning digital interference holography (WSDIH) for tomographic imaging,[5] or multiwavelength metrology,[6] where the wave field (amplitude or phase maps) at different positions, from different wavelengths, or even from different cameras needs to be compared or combined. The same requirement arises in variable tomographic scanning,[7] where the object field is reconstructed in a number of selected tilted planes from a series of holograms recorded with different wavelengths, and the numerical superposition of all the tilted object fields results in a short coherence length.

The Fresnel diffraction formula (FDF) is popularly used in DH; however its pixel resolution will increase in proportion to the reconstruction distance. The Fresnel approximation condition requires an object-to-hologram distance large enough to guarantee precise reconstruction, but the FDF may work well even if the approximation condition is not strictly applied.[5] However, in numerical implementation the FDF is also subject to a minimum object-to-hologram distance limitation (or $z_{\min}$ requirement): otherwise, aliasing occurs. The $z_{\min}$ requirement must be guaranteed in any case in the FDF, and $z_{\min}$ is the distance at which the reconstructed plane has the same resolution as the hologram. Since it is such an important characteristic for the FDF and its resolution analysis, the $z_{\min}$ requirement is the main concern in this Letter. The FDF can also be implemented as a convolution, which can be used within the $z_{\min}$ distance. However, the convolution method (CM) does not work if the reconstruction plane is close to the hologram plane.[5] Note that the CM here means the convolution implementation of the FDF; some researchers may define the CM as a form similar to the angular spectrum method[1] (ASM); however, both the CM and the ASM fix the pixel resolution at that of the CCD camera. A zero padding method[8] was proposed to control the resolution for the FDF, where the new resolution is decreased by adding more zeros to increase the total pixel number. However, this method cannot be used to adjust the pixel resolution for a distance smaller than $z_{\min}$. A double-Fresnel-transform method[9] (DFTM) was also recently proposed to adjust the reconstruction pixel by introducing a transitional plane (TP) and implementing the FDF twice. The final resolution is proportional to the ratio $|z_2|/|z_1|$, where $|z_2|$ is the distance from the TP to the destination plane (DP) and $|z_1|$ is the distance from the hologram to the TP, with $|z_1|, |z_2| \geqslant z_{\min}$. If the object-to-hologram distance is small, the above ratio can be adjusted only in a limited range. Specifically, the DFTM can not be used for resolution control if the DP is close to the hologram. In this Letter we propose an alternative approach to control the pixel resolution in digital holography. The wave field on either a variable tilted or vertical plane can be reconstructed with adjustable resolution, and the reconstruction distance can be any small distance without being subject to the $z_{\min}$ limitation, which is a unique capability not available in any other existing reconstruction algorithms.

If the reconstruction distance is large enough that the Fresnel approximation condition can be assumed, the wave distribution on a variable tilted x_o-y_o plane, with its normal tilted at an angle θ in the $y-z$ plane as in Fig. 1(a), can be calculated as[7]

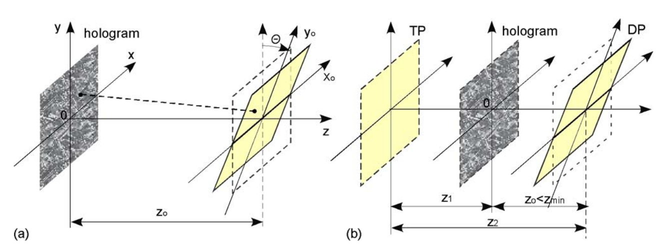

Fig. 1. (Color online) (a) Reconstruction of the wave field on a tilted destination plane (DP); (b) resolution control by introducing a transitional plane (TP).

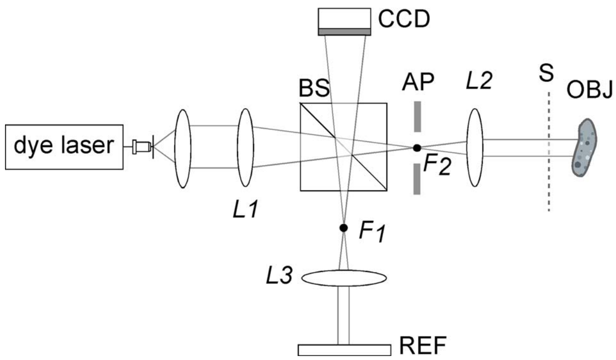

Fig. 2. Apparatus for the digital interference holography system.

$$E(x_o,y_o,z_o) = \frac{iE_o}{\lambda}\exp\left[ik\left(r_o + \frac{z_o y_o \sin\theta}{r_o}\right)\right] \times \int\int o(x,y)\exp\left[\frac{ik}{2z_o}(x^2+y^2)\right] \times \exp\left[-\frac{ik}{r_o}(x_o x + y_o y\cos\theta)\right]\mathrm{d}x\mathrm{d}y, \quad (1)$$

where $o(x,y)$ is the object wave information extracted from the hologram plane at the $z=0$ plane, k is the wavenumber given by $k=2\pi/\lambda$, E_0 is a constant, and $r_o=(z_o{}^2+x_o{}^2+y_o{}^2)^{1/2}$. In the discrete implementation of Eq. (1), the resolution of the reconstructed plane (x_o,y_o) is determined as

$$\Delta x_o = \left|\frac{\lambda z}{N\Delta x}\right|, \qquad \Delta y_o = \left|\frac{\lambda z}{N\Delta y\cos\theta}\right|, \quad (2)$$

where Δx_o and Δy_o are the resolutions of the tilted plane, Δx and Δy (equal to Δx) are the resolutions of the hologram plane, and $N\times N$ is the array size of a square area on the CCD. Equation (1) is called the tilted Fresnel diffraction formula (TFDF) in this Letter. Note that if the tilted angle θ is equal to zero, then it becomes the well-known FDF. In particular, aliasing occurs during numerical implementation if $|z|<z_{\min}=N(\Delta x)^2/\lambda$, which sets the minimum object-to-hologram distance.

However, a small object-to-hologram distance is preferred in some optical systems. For example, a system can be made more compact, or as in a WSDIH system, tomographic images with higher signal-to-noise ratios can be achieved if the object is close to focus. Thus the above algorithm cannot be directly used for reconstruction. In order to solve this problem, a transitional reconstruction plane is introduced, and the wave field on the TP is reconstructed by use of the ASM, which has the great advantage of reconstructing wave fields close to the hologram plane, even at distances down to zero. First, the object angular spectrum at the hologram plane, $S(k_x,k_y;0)$, is obtained by taking the Fourier transform of the object wave $o(x,y;0)$, where k_x and k_y are corresponding spatial frequencies of x and y. The TP is introduced opposite to the DP on the z axis, as shown in Fig. 1(b). The angular spectrum of the TP (at z_1), $S(k_x,k_y;z_1)$, can be calculated as $S(k_x,k_y;0)\exp(ik_z z_1)$, with $k_z=(k^2-k_x^2-k_y^2)^{1/2}$. Finally, the complex wave field on the TP, $o(x,y;z_1)$, can be calculated from the inverse Fourier transform of $S(k_x,k_y;z_1)$. The resolution of the reconstructed TP is also Δx, the same as that of the hologram plane.

Second, the wave distribution in the tilted (or vertical) DP is reconstructed directly from the TP by use of Eq. (1), and the pixel resolution at the DP is given as

$$\Delta x_o = \left|\frac{\lambda z_2}{N\Delta x}\right| = \left|\frac{z_2}{z_{\min}}\right|\Delta x, \qquad \Delta y_o = \frac{\Delta x_o}{\cos\theta}, \quad (3)$$

where $z_2=z_o-z_1$ is the distance from the TP to the center of the DP and $z_{\min}=N(\Delta x)^2/\lambda$ as defined above; thus the pixel resolution can be easily adjusted by selecting a proper z_1 for the TP. Note that z_1 is normally selected to satisfy $|z_2|\geqslant z_{\min}$. However the distance from the DP to the hologram, or the original object-to-hologram distance $|z_o|$, can be any small distance without being limited by the minimum distance requirement. Of course, if the original object-to-hologram distance $|z_o|$ is larger than $z_{\min}$, the TP can also be placed on the positive z axis. Theoretically, the new resolution Δx_o can be any value greater than Δx.

Experiments are performed to verify the effectiveness of the proposed algorithm. Figure 2 shows an off-axis digital holographic setup based on a Michelson interferometer.[7] The collimated plane wave from a Coherent 699 ring dye laser is focused by lens L1 onto the focal point F1 or F2. Point F2 is also the front focus of objective L2, so the object is illuminated with a collimated beam. Plane S is imaged to the CCD camera by lens L2. In the reference arm the beam is also collimated by lens L3, which results in a magnified image at the CCD camera of an interference pattern that would exist at S if the object wave were superposed with a plane wave there. Aperture AP is placed in the focal plane of L2 to control the size of the object angular spectrum captured in the CCD camera.

In our experiment the system images a surface of a 25 cent coin, containing three letters "IBE" within a 2.5 mm × 2.5 mm area of 300 × 300 pixels; thus the resolution of the hologram is 8.3 μm. The coin is slightly tilted with a small angle $\theta=4°$ to the hologram plane. The wavelength of the dye laser is 580 nm. The reconstruction distance z_o, representing the distance from the object to plane S in Fig. 2 is about 0.1 mm. In order to use FDF for reconstruction, the $z_{\min}$ required for the system is 35.9 mm, which is much larger that the actual $z_o=0.1$ mm in the setup. The reconstruction results of Figs. 3(a) and 3(b) clearly show that neither the FDF nor the CM works in this case. Obviously, the zero padding method does not work either, since it is based on the FDF and subject to the $z_{\min}$ requirement. A direct reconstruction from the ASM gives a proper result, as shown in Fig. 3(c). However, the reconstructed pixel

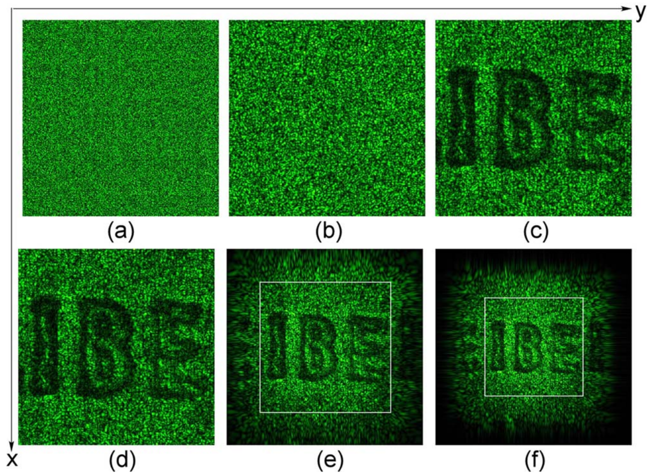

Fig. 3. (Color online) Reconstruction from the (a) FDF, (b) CM, (c) ASM, and (d) DFTM; (e) and (f) are the reconstructions of the proposed algorithm with the pixel resolutions Δx_o, Δy_o equal to 12.5 and 17.7 μm, respectively.

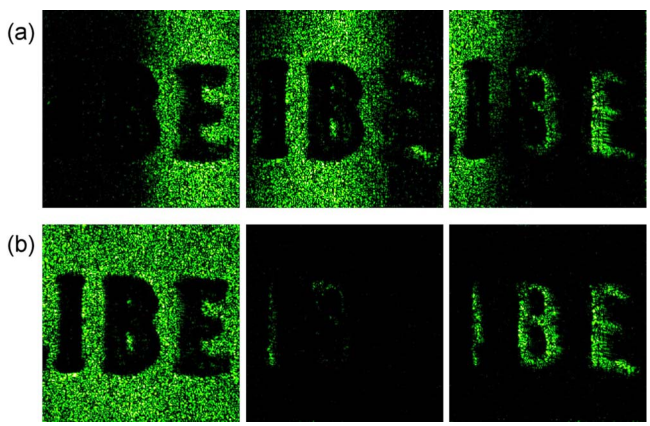

Fig. 4. (Color online) Contour images of the coin at 60 μm axial distance intervals with (a) $\theta=0°$ and (b) $\theta=4°$ in reconstruction.

resolution in both directions is fixed as 8.3 μm and cannot be adjusted as in the CM, and the reconstruction planes are all parallel to the hologram plane. For a small object-to-hologram distance as above ($|z_0| < z_{\min}$), the DFTM can be used for resolution control with a scheme similar to that in Fig. 1(b) but requires $|z_1| \geq z_{\min}$, and the new resolution is given as $(1+|z_0/z_1|)\Delta x$. Obviously, even if z_1 is not at infinity, but if $|z_0|$ is small, the DFTM still cannot be used for resolution control. Figure 3(d) shows a reconstruction by the DFTM with the TP placed at $z_1=-35.9$ mm; one can hardly see any resolution difference between Figs. 3(d) and 3(c).

With the algorithm proposed in this Letter, the pixel resolution can be easily adjusted by changing the position of the TP. For example, if the TP is introduced at $z_1=-53.8$ mm and the rotation angle is set to be $\theta=0°$, the reconstructed image is shown as Fig. 3(e) with Δx_o, Δy_o equal to 12.5 μm. Figure 3(f) shows another reconstruction with Δx_o, Δy_o equal to 17.7 μm when $z_1=-71.7$ mm is used. Since either the hologram or the TP is a sampled lattice, nonoverlapping higher-order terms of diffraction may appear in a DP of larger resolution, as shown in Figs. 3(e) and 3(f). The rectangle in the figure shows the first-order reconstruction, which is of the main interest and can be easily extracted, since its image size is determined by the new resolution and its position can be precisely controlled by the shift of the angular spectrum. Of course, if the conjugate spectrum of the object is not completely filtered out for off-axis holography, a portion of the conjugate image will also appear as a residue in the reconstruction.

A direct application of the proposed algorithm is in WSDIH. For the same object at $z_o=0.1$ mm as above, for example, if the above process is repeated by using 11 different wavelengths from a range of 580.0 to 585.0 nm, and all the reconstructed wave fields are overlapped with the same pixel resolution of 8.3 μm, tomographic images can be achieved with a 60 μm axial resolution according to Ref. 7. Figure 4(a) shows several contour images parallel to the hologram plane, since $\theta=0°$ is used in the algorithm. Figure 4(b) shows the contour images when the reconstruction planes are tilted with $\theta=4°$ in reconstruction. One can clearly see that the letters on the coin are now either all highlighted or all darkened, for they are located in the same scanning plane. Note that since the object distance $|z_o|$ is so small compared with $z_{\min}$, it is impossible to use any other available algorithms directly for tilted tomographic reconstruction. However, with the proposed algorithm variable tomographic scanning is possible, and the pixel resolution can be easily adjusted.

In conclusion, we have shown that wave fields on a tilted plane (or a vertical plane) can be reconstructed near the hologram plane without being subject to the minimum object-to-hologram requirement, and the pixel resolution can be easily controlled. The proposed algorithm would be extremely useful for WSDIH, multicolor holograms, and metrological applications where wave fields of different resolutions need to be compared or combined. It makes pixel resolution control possible, especially when a small object-to-hologram distance is preferred in the system.

This work is supported in part by the National Science Foundation. L. Yu's email address is yulingfeng@gmail.com, and he gratefully acknowledges the financial support of the Alexander von Humboldt Foundation of Germany.

References

1. U. Schnars and W. Juptner, Meas. Sci. Technol. **13**, R85 (2002).
2. P. Ferraro, G. Coppola, S. De Nicola, A. Finizio, and G. Pierattini, Opt. Lett. **28**, 1257 (2003).
3. I. Yamaguchi, T. Matsumura, and J. Kato, Opt. Lett. **27**, 1108 (2002).
4. J. Kato, I. Yamaguchi, and T. Matsumura, Opt. Lett. **27**, 1403 (2002).
5. L. Yu and M. K. Kim, Opt. Lett. **30**, 2092 (2005).
6. J. Gass, A. Dakoff, and M. K. Kim, Opt. Lett. **28**, 1141 (2003).
7. L. Yu and M. K. Kim, Opt. Express **13**, 5621 (2005).
8. P. Ferraro, S. Nicola, G. Coppola, A. Finizio, D. Alfieri, and G. Pierattini, Opt. Lett. **29**, 854 (2004).
9. F. Zhang, I. Yamaguchi, and L. P. Yaroslavsky, Opt. Lett. **29**, 1668 (2004).